CHAPTER V

ON THE TRANSFORMATION BETWEEN TRANSIT DISTANCES AND WIND COMPONENT DISTRIBUTIONS

by

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V-1. INTRODUCTION

The recent conceptual development of the crossed-beam concept indicates that the correlation of A.C.-coupled photometer outputs can be employed to approximate the probability distribution of either transit times or transit distances between spatially and temporally separated lines of sight. Applications of these measurements then require us to relate these transit times or distances to the velocity components associated with the transport of heat, mass, or momentum. The objective of this memorandum is to extend this transformation to the use of transit distances instead of transit times.

Present experiments have been restricted to the use of transit times. These transit times may be uniquely related to wind components, if the direction and length of the transit distance are known. In wind tunnel and jet applications, the transit distance is set by separating two mutually perpendicular beams along the known flow direction [V-1]. Subsequent transformation from transit times to longitudinal velocity variations has successfully matched pitot tube and hot-wire measurements. The early field tests of ground winds at tower heights [V-2] employed an outside wind direction reading with a wind vane to obtain the transit distance. The wind direction at the top of the tower was measured before the test run, and both beams were then rotated normal to this direction. The beam separation at the tower height is then a good estimate of the transit distance provided that the wind direction fluctuations during the run do not exceed ±20 degrees [V-3].

Analytical studies of multibeam geometries were initiated to determine whether suitable test geometries could be found which do not require independent wind direction readings for relating transit times to wind

components. W. H. Heybey [V-4] in his investigation of three ground telescopes showed that the two horizontal wind components could be determined at preselected altitudes, provided that vertical wind components are negligible. A recent extension of his work [V-5] also indicates that the third telescope may be replaced with one additional photodetector in the focal plane of the second telescope. This simple single-beam fan geometry will measure horizontal winds over a restricted range of wind azimuths ($\approx \pm 22°$). Vertical wind measurements are beyond the capability of both the single-beam fan arrangement and the three-telescope arrangement. They would require the use of at least four ground-based telescopes.

The purpose of this paper is to present physical arguments that wind components may be uniquely determined with a simple two-telescope arrangement if a plurality of photodetectors is used in the focal plane of one telescope to monitor transit distances instead of transit times. The use of a third or fourth telescope and the restrictions on wind azimuth are not necessary, if the measurement objectives are to measure one wind component rather than two or three.

V-2. THE MEASUREMENT OF TRANSIT DISTANCES

Consider the test geometry of Figure V.1. One distant radiometer, B, is set to look at almost horizontal lines of sight which are all contained in the same vertical plane. A second radiometer, A, is set normal to this plane such that its horizontal line of sight intersects the lowest beam of the distant telescope, B. The fan angles between the other lines of sight of radiometer B are assumed to be so small that deviations from the horizontal may be neglected in the vicinity of the intersection point. Each line of sight of radiometer B may thus be characterized by its altitude $z = h, h_2, h_3, \ldots$, above the intersection point. Transit distances are measured with the above arrangement by differentiating the output signals $i_A(t)$ and $i_B(t,h)$ of the two radiometers by delaying signal i_A relative to the signals i_B, and by adding the lagged product of these time derivatives. The result of this cross-correlation calculation is then multiplied with the selected time delay and normalized with the resultant peak value as shown in Figure V-2. Plotting this normalized multibeam product mean value against the abscissa h/τ should then directly approximate the probability $P(w = h/\tau)$ that the vertical speed w of optical disturbances has the value h/τ.

The above claim of approximating vertical wind speed variations is based on the same physical model of the common path that was used by Heybey. The common signals, which are retrieved by the correlation calculation, are identified with optical disturbances which intersect beam A at time $t - \tau$ and B at time t. These disturbances therefore

travel along a "common path" that intersects beam A and beam B at the altitude h above beam A. Let us assume that the vertical wind component, w, does not change along the common path. A disturbance can then be "common" to the altitudes z = 0 and z = h only if the vertical transit distance component $w\tau$ is equal to the given altitude h.

$$w = h/\tau. \tag{1}$$

The time history of horizontal wind component, v, will determine the displacement of the common path intersection of beam A relative to the beam intersection point at z = 0. This displacement distance is given by

$$\int_{t-\tau}^{t} v \, dt.$$

Let us now assume for a moment that the turbulence is homogeneous over such distances. The common signals which are generated at this intersection are then statistically equivalent to the disturbances for any path intersection point on beam A. In other words, the displacement of the path intersection along beam A does not change the product mean value calculations. The magnitude of variations of the horizontal wind components v are thus suppressed in the product mean value calculations. Any displacement of a common disturbance along the beam cannot be detected because of the optical integration along the beam. The above arguments may be repeated for the displacement

$$\int_{t-\tau}^{t} u \, d\tau,$$

of the common path intersection along beam A at altitude z = h. As a result, we thus find that any horizontal motion is cancelled by the optical integration along the beams in fields of homogeneous turbulence. These same arguments should hold, in first approximation, for inhomogeneous fields of turbulence, where the linear dependence of point-beam product mean values on translational disturbances

$$\int v \, dt \quad \text{or} \quad \int u \, dt$$

is cancelled by the subsequent integration over all such displacements.

The above arguments are completely general in that they do not depend on the special orientations of the beams B or beam A. The arrangement of horizontal beams that is illustrated in Figure V-1 was chosen to demonstrate that vertical wind components can be obtained with only two telescopes. The same arrangement would also closely simulate the desired airplane instrument-package [V-6]. However, the particular crosscorrelation calculations given in Figure V-2 will indicate transit distance components, h, across the fan regardless of orientation of the fan normal. For example, horizontal wind components would be measured if the beam configuration of Figure V-1 is rotated around their common baseline until the fan beam normal is in a horizontal position. In any case, the crosscorrelation should indicate the probability of normal wind components across the fan of beams B

$$P(w = h/\tau) = \text{const}\ \frac{\tau}{h} < \frac{\partial i_A(t-\tau)}{\partial t}\ \frac{\partial i_B(t,h)}{\partial t} > . \qquad (2)$$

The transformation of these transit distances to wind components is already included in equation (2). This transformation no longer requires an independent measurement, since the transit time is equal to the time delay τ = const., which was set by the computer.

V-3. ACCEPTABLE RANGE OF TIME DELAYS

The above discussions would be incomplete without considering the restriction on the acceptable range of time delays. A lower limit for the time delay, τ_{min}, is given by the condition that the associated transit distance, $\bar{w}\tau$, should exceed the class interval of its probability distribution. This class interval is given by the length scale of the common disturbance, and was reduced to the turbulent microscale, L_h, by correlating the signal derivatives $\partial i/\partial t$, instead of the signals [V-7].

$$L_h = \frac{\bar{w}}{2\pi} \sqrt{\frac{< i_A(t)\ i_B(t;\ h=0) >}{< \frac{\partial i_A(i)}{\partial t}\ \frac{\partial i_B(t,0)}{\partial t} >}} \ . \qquad (3)$$

The lower limit of transit distances is thus provided by the condition:

$$\frac{L_h}{\bar{w}\tau_{min}} = \frac{1}{\tau_{min}} \sqrt{\frac{< i_A(t)\ i_B(t;\ h=0) >}{< \frac{\partial i_A(t)}{\partial t}\ \frac{\partial i_B(t,0)}{\partial t} >}} \ll 1. \qquad (4)$$

An upper limit of the time delay, τ_{max}, is given by the condition that the transit time should be much smaller than the eddy lifetime, L_τ. A first approximation of this eddy lifetime may be derived from the envelope to the various time correlation curves, as illustrated in Figure V-3.

$$\tau_{max} \leqq L_\tau.$$

Both inequalities together define the range of acceptable time delays:

$$\frac{1}{2\pi} \sqrt{\frac{< i_A(t)\ i_B(t;\ h=0) >}{< \frac{\partial i_A(t)}{\partial t}\ \frac{\partial i_B(t;0)}{\partial t} >}} \ll \tau \ll L_\tau. \qquad (6)$$

Within this acceptable range of transit times, one should expect that the crossed-beam approximation of statistical wind-component distributions, equation (2), is valid. One may thus verify this approximation by repeating the crosscorrelation calculations for various time delays, $\tau = \tau_1; \tau_2; \tau_3 \ldots$, within the acceptable range. The resulting points should all collapse on one curve. The same condition may also be used to obtain a sufficient number of points from comparatively few (≈ 5) beams B. This additional advantage is also illustrated in Figure V-2 for two selected time delays, $\tau = \tau_1$ and $\tau = \tau_2$. The same procedure might be used to develop an on-line wind component display system.

V-4. ACCEPTABLE RANGE OF FAN ANGLES

An arrangement of photodiodes in the focal plane of telescope B will give a fan of narrow conical fields of view. These fields of view replace our hypothetical parallel beams in an actual photodetection system. We

therefore have to consider a finite range of fan angles, B, and field of view, $\Delta\beta$. The previous idealization of parallel beams may be considered as the limiting case, where the fan angles all approach zero.

The effects of a finite fan angle, $\beta > 0$, are illustrated in Figure V.4. Consider the horizontal displacement,

$$\tau \sqrt{\bar{u}^2 + \bar{v}^2},$$

across a single photodiode's field of view. The normal displacement, h, of a common signal will be subject to a geometrical uncertainty,

$$\frac{1}{2} \tau \tan \beta \sqrt{\bar{u}^2 + \bar{v}^2},$$

due to the finite fan angle. This uncertainty should be smaller than the tolerance, L_h, of the normal displacement distance, h. This gives the following geometrical condition for the acceptable range of fan angles, β:

$$\frac{1}{2} \tau \tan \beta \sqrt{\bar{u}^2 + \bar{v}^2} \leq L_h. \tag{7}$$

The effects of a finite viewing angle, $\Delta\beta$, will be to generate a finite beam diameter, $h\Delta\beta/\sin\beta$, which should also be smaller than the tolerance on transit distance. This leads to a geometrical condition for the acceptable field of view.

$$\frac{h \Delta\beta}{\sin \beta} \leq L_h. \tag{8}$$

However, the ratio $n = h/L_h$ gives the number of beams in the fan. Equation (8) may thus be rewritten as

$$\Delta\beta = \frac{1}{n} \sin \beta. \tag{9}$$

Equation (7) becomes

$$\tan \beta \leqq \frac{2L_h}{h} \frac{h}{\tau \sqrt{\bar{u}^2 + \bar{v}^2}} = \frac{2}{n} \frac{\bar{w}}{\sqrt{\bar{u}^2 + \bar{v}^2}} \quad . \tag{10}$$

In case of vertical wind measurements, $\bar{w}/\sqrt{\bar{u}^2 + \bar{v}^2}$ has values of ≈ 0.1 and the largest acceptable fan angle of a five-beam fan would be

$$\tan \beta = \beta \leqq \frac{2}{5} \cdot 0.1 \approx 2°.$$

The associated viewing angle would be

$$\Delta\beta \leqq \frac{2}{5} = 0.4°.$$

Such a system can obviously be achieved by installing a variable-fiber optics block in the already existing detector fan.

V-5. CONCLUSIONS

A plurality of no more than five photodetection elements in the focal plane of a single telescope is proposed to monitor wind components in preselected directions by monitoring transit distances instead of transit times. Physical arguments are presented which lead to the following tentative conclusions.

(1) Any component of the atmospheric motion along one of the beams will not affect the result of crosscorrelation calculations, since a displacement of a common disturbance along a beam cannot be detected.

(2) The crosscorrelation calculations indicate only the components of the transit distances which are normal to both beams.

(3) The measured transit distances can be directly transformed into wind components, since the transit time is known from the time delay, which was set in the computer.

(4) Both horizontal and vertical wind components can be determined using no more than two telescopes.

(5) The above wind component measurements require us to approximate the probability density of wind component variations for various time delay parameters. The range of these parameters may be determined experimentally with crossed-beam systems.

(6) A plurality of photodetectors will always give a finite fan angle and a finite viewing angle; whereas, the analysis was based on the idealized case of a vanishing fan angle. However, because of the finite size of the turbulent microscale, small fan angles and viewing angles, which are within the capability of the already existing detector fan, are tolerable.

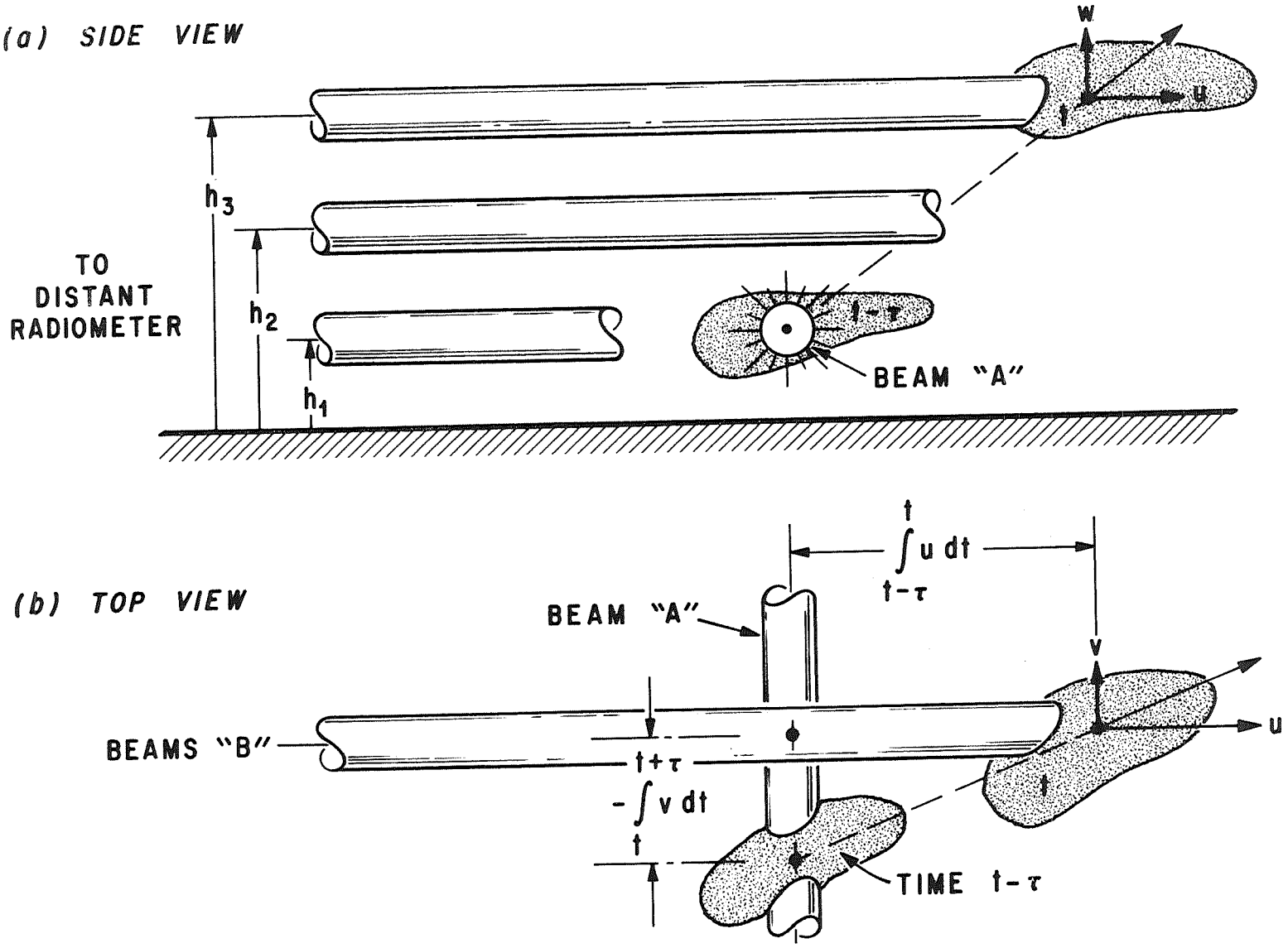

FIGURE V.1. CROSSED BEAM TEST ARRANGEMENT FOR TRANSIT DISTANCE MEASUREMENT

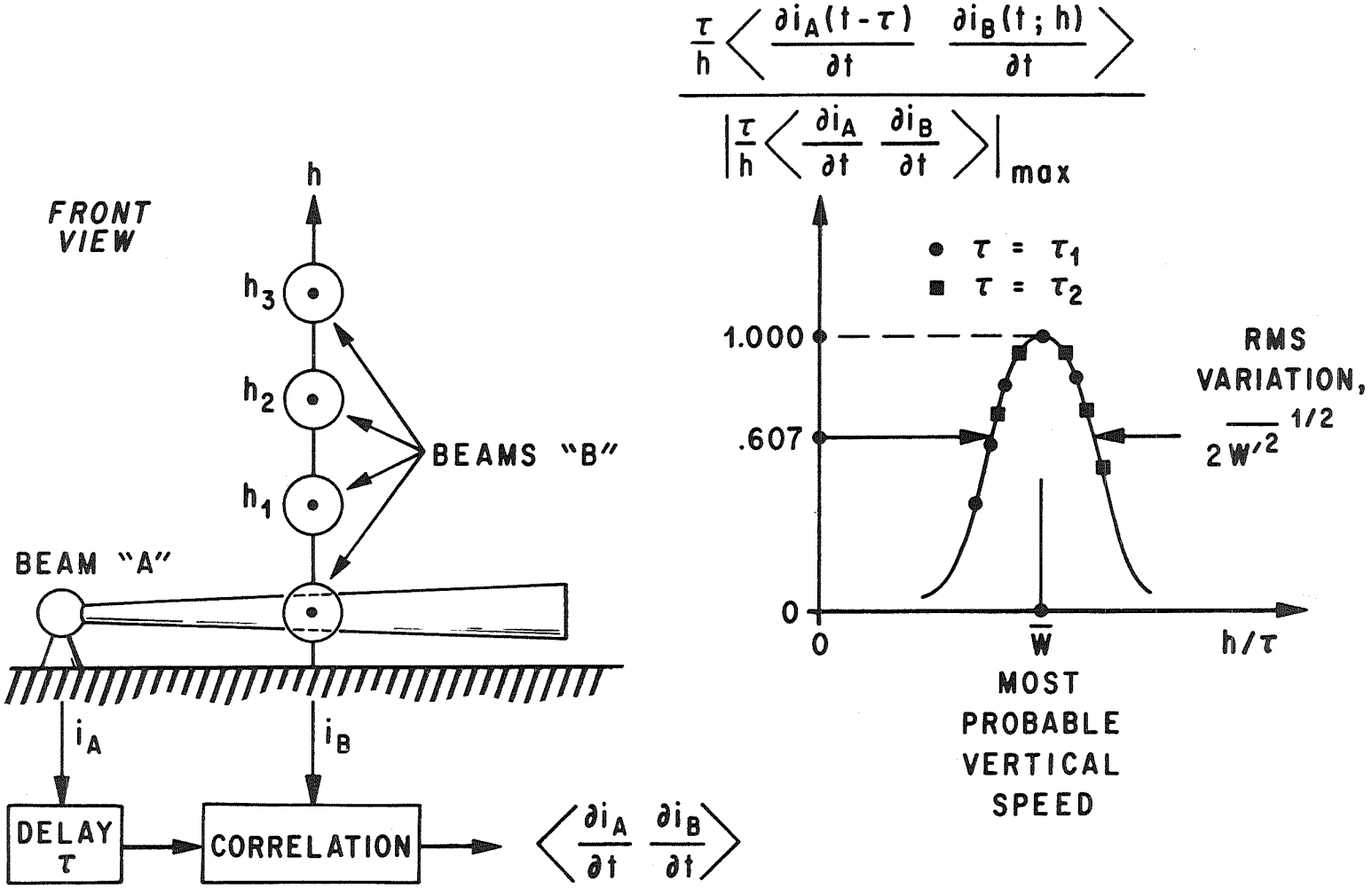

FIGURE V.2. PROBABILITY DENSITY OF WIND COMPONENT VARIATIONS

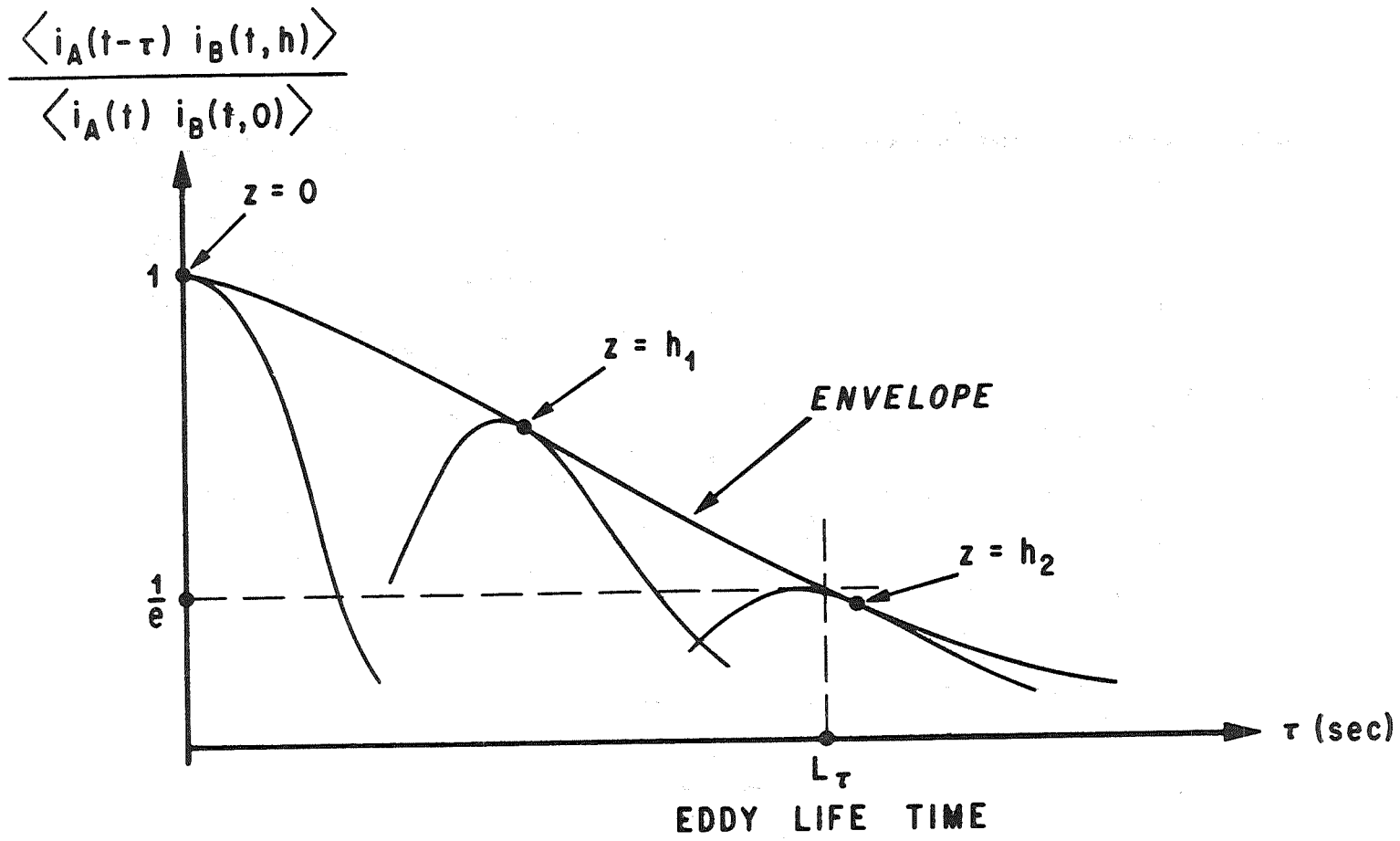

FIGURE V.3. APPROXIMATION OF EDDY LIFETIME

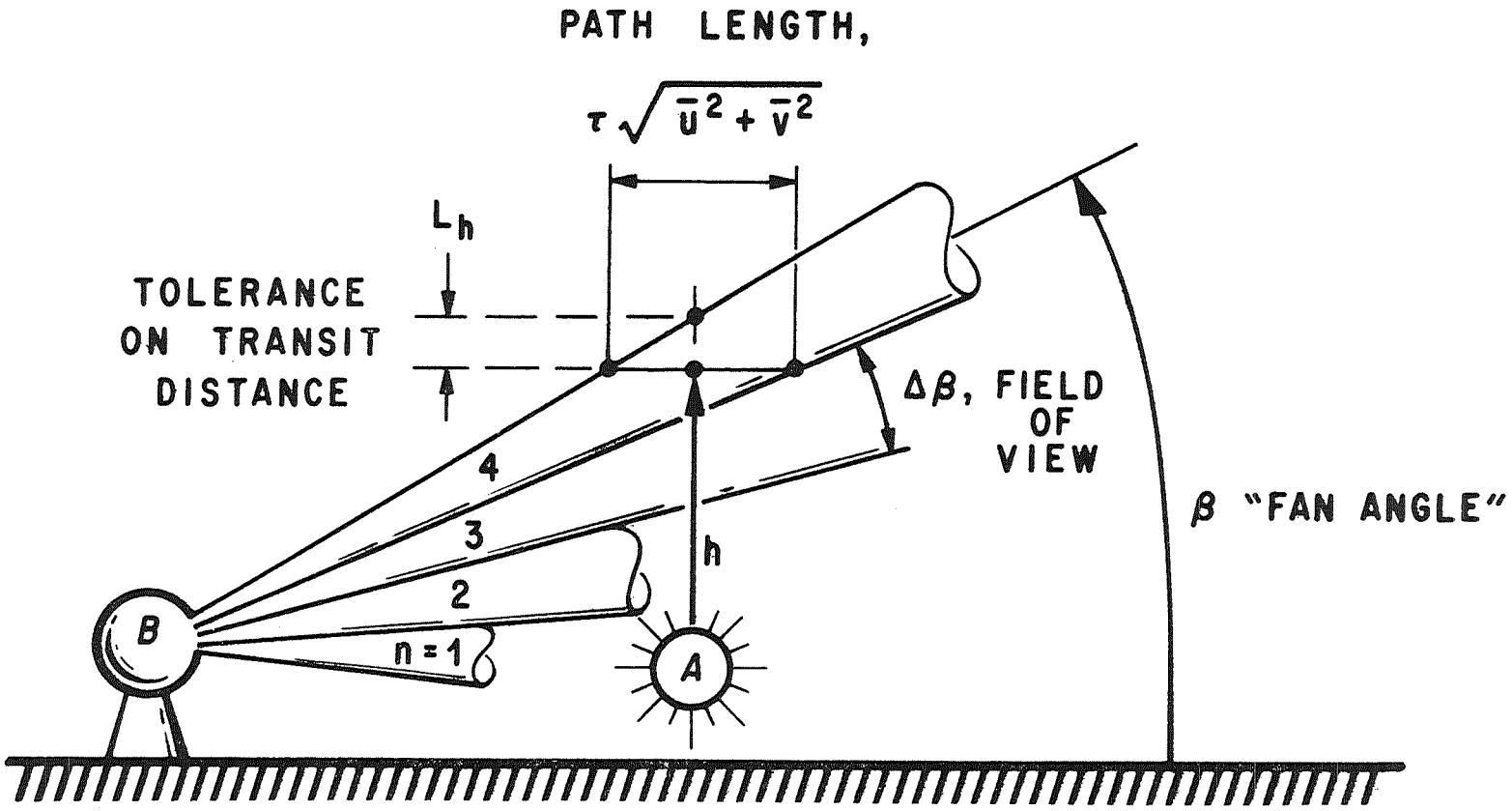

FIGURE V.4. ACCEPTABLE RANGE OF FAN ANGLES

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